Skin Graph-t June 22

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We are particularly proud of our solutions in: 6/9

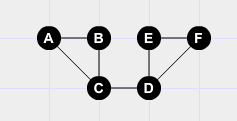
Lots of good work today! These were hard problems, and you came up with some great ideas. Be sure to include pictures in your write-up next time when appropriate; graph theory is a very visual subject. Also, be clear about what you can prove, and what you think is true but can’t prove (yet).--RK

Morning Report: IN class, we first discussed the solutions to the graph with order 6 and size 12 that does not contain K4. We then defined the degree of a vertex, degree sequences, connected and disconnected graphs, general graphs, subgraphs, degree sums, the paths between vertices, and the diameters and distances between vertices. We then discussed several of the problems that we solved during our class-time, with Kevin arguably discussing the most. In our group we discussed ways in which to write a proof for problem 6 from the first in-class worksheet. In our proof we proved the opposite of the question to be false. If no one person knows an even number of people, then the degree sum will necessarily be odd as the addition of an odd number (51) of odd integers is odd. This contradicts the fact that the degree sum, 2n, must always be even. Nice report. Make sure you all find a way to rock on in the mornings! -DJS

Afternoon Report:

We decided that in the afternoon, we would solve the problems on our own, and then share and debate afterwards. Kevin finished first and worked with the helpers on question 6/9, ending up being able to solve the question from a different perspective. Kevin missed some debating time afterwards, but did consult some with the rest of the group. Kevin and Mitchell had a disagreement over the role of group members after Kevin had finished his problems. Mitchell argued that Kevin should inform the group of his discoveries on 6 and 9 after he had finished all 10 of his problems. The rest of the group (Dean and Mitchell) was in the process of finishing, but had not finished the entirety of their problems and were in need of assistance. Kevin refused to assist, arguing that the other two group members should solve it on their own. This caused a difference of opinion, that is still a point of conflict, between Mitchell and Kevin. Additional problems from the group included minor mathematical differences between the three group members which were resolved between them with healthy and polite mathematical discussion. The problems were relatively few and far between, due to the fact that we worked independently from the beginning and reconvened later. I’m reading what you are saying both in plaintext and in subtext. I’m hoping that the three of you find a good way to work together. We’re going to mix the groups up after Wednesday’s class, so people will get to experience a variety of different teammates. I’ll announce tomorrow that people are meant to be working and sharing info as we go. DJS

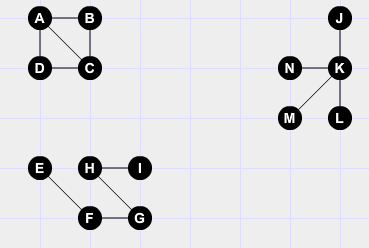
0. Group Name:The Skin Graph-t

1. Find a graph G with order 6 and size 7 such that G contains no 4-cycles
   1. 4-cycles are four vertices interconnected (as in a path)
   2. 

* Good job on finding one example, can you find more next time? - Lizzy

1. find all (nonisomorphic) graphs where (order) + (size) = 8
   1. 1-8 vertices
   2. 1-7 vertices with one edge
   3. 2-6 vertices with two edges
   4. 3-5 vertices with two edges
   5. 4-4 vertices with four edges
   6. =9 total solutions

* Next time, try drawing out the solutions that you found, also the format of the solutions you have are a little confusing. This solution is not correct, and not written properly. Lizzy
* I know your group worked hard on this problem, but just giving the numbers of vertices and edges isn’t enough information to know which graph you’re talking about. Can you see why?--RK

1. Find all (nonisomorphic) connected graphs as you can with the property that (order)(size)=20
   1. possible combinations of orders and sizes
      1. 1-20
      2. 2-10
      3. 4-5
      4. 5-4
      5. 10-2
      6. 20-1
   2. because most of these combinations are not possible or cannot create connected graphs, the only combinations possible are 4-5 and 5-4
   3. there are 3 graphs for 5 order and 4 size, again, show us your examples, also make sure that the examples that you found are not isomorphic. -Lizzy
   4. there are 2 graphs for 4 order and 5 size
   5. =5 possible solutions you have only drawn ⅗ solutions you claim to have found. -Lizzy
2. A graph is *n-regular* if all vertices are of degree n. How many 3-regular graphs are there of order 11? Prove that you have found them all.
   1. due to the degree sum’s quality of being even, an odd number (11) of odd number degrees is not possible.

This is a solid argument, stated very concisely. Well done!--RK  
The distance between two vertices is the length of the shortest path between them. The diameter of a graph is the longest distance between two vertices.

* 1. Draw a graph with order 5 and diameter 1
  2. Draw a graph with order 5 and diameter 2
  3. Draw a graph with order 5 and diameter 3
  4. Can you categorize the graphs of diameter 1? Diameter 2?

No answer? - Lizzy

1. Find a 3-regular graph of diameter 2 with the largest possible order.
   1. At first, we believed that the largest possible order was 8, using diameters (as if an octagon was inscribed in a circle). Afterwards, with some support and help, Kevin found that a graph of order 10 was also possible, using 2 layers of five, the inner layer forming a star and connecting to a vertex on the outer layer, and the outer layer creating a pentagon. We hypothesize that this is as far as we can take the order, as in question five, we found that the simplest graph with order 5 and diameter 2 would be a simple pentagon, thereby suggesting that an order of 10 (5 times 2) would be the maximum, (in a similar format of 2 layers, 12 would have too long of a route from one vertex to another).

This graph does work for order 10. Congratulations on solving a hard puzzle! Your solution would be much clearer with a picture; without context, it’s not clear what you mean by “layers.” By the way, it is possible to prove that 10 is the maximum order.

To see why, try thinking about this question: in a 3-regular graph, what is the maximum number of vertices which can have distance 2 from a single vertex?--RK

1. Prove that, for every lovely party you host, the number of people who shook an odd number of hands is even.
   1. [Draw a graph of lovely party graphs,assume that the those who shook hands are an edge between party guests]The degree sum must be even. Having an odd number of people who shook an odd number of hands would force the degree sum to be odd. Therefore the number of people who shook an odd number of hands must be even.

Next time, when you write a proof try to set up the basis of the problem within your answer. It makes your solution sound better, and is more professional.- Lizzy

Your argument is great here! You just need a sentence or two explaining the connection between the problem (which talks about parties and handshakes) and your solutions (which talks about degree sums.) What graph are you drawing and why? What does a vertex represent? What does an edge represent?--RK

1. A cycle is like a path, except if begins and ends at the same vertex. Let G be a graph. Prove that if every vertex of G has a degree of at least 2, then G contains a cycle.
   1. All points in a cycle have a degree of at least 2, thereby creating a so-called “path” leading from one point to another coming back to the original point.

I’m not sure if this is what you meant, but what you argued here is that every vertex in a cycle has degree at least 2. This is true by definition! What you need to show is something different: that if you have any graph, and each vertex has degree 2, then the graph must contain a cycle. Does this make sense?--RK

1. Find the largest (greatest order) graph with a maximum degree 3 and diameter 2
   1. see question 6

This graph does work! Can you prove it? (See my comment on problem 6.)--RK

ARE there two (non isomorphic graphs with order 6 and size 12 that do not contain K4 as a subgraph?

* 1. no. we could not find a second graph that fulfilled the constraints.

I think you’re right (I haven’t yet proved it myself). Can you prove it? Not finding an example is good evidence, but it’s not a proof all by itself.--RK

1. UNSOLVED! The Erdos-Gyarfas conjecture: Every graph with minimum degree 3 has a cycle whose length is a power of 2. It is certainly true for every such graph that *I* have drawn. Can you prove that it is always true? Can you find a counter example? G’head and try!
   1. did not have time to consult